DOI: 10.18698/1812-3368-2018-2-60-72

A NEW LOOK AT FUNDAMENTALS OF THE PHOTOMETRIC LIGHT TRANSPORT AND SCATTERING THEORY. PART 3: BRIDGES TO MULTI-DIMENSIONAL PROBLEMS

S. Persheyev[1] sp95@st-andrews.ac.uk
D.A. Rogatkin[2]

[1] **School of Physics and Astronomy, St Andrews University, St Andrews, The United Kingdom of Great Britain and Northern Ireland**
[2] **Moscow Regional Research and Clinical Institute named after M.F. Vladimirsky, Moscow, Russian Federation**

Abstract

In previous two parts of the article, one-dimensional (1D) scattering processes were taken into detailed consideration. All main typical 1D scattering problems of different complexities were discussed and solved using different approaches. It gave the opportunity to find ways to improve the theory, two-fluxes Kubelka — Munk approach, in particular. It was shown that scattering and absorption processes inside the light-scattering medium are not independent, so the formulation of first coefficients of transport differential equations as the simplest sum of scattering and absorption coefficients is wrong. Inaccuracy in this formulation leads to inaccuracies in results. In this final part of the article, as a completion, the analysis of some spatial light-scattering problems, mainly two-dimensional (2D) problems as the simplest multidimensional problems for consideration, is presented. The detailed analysis of several important 2D approximations, such as a pure backscattering approximation, single-scattering one for a pencil-like beam, and an orthogonal-scattering approach opens the way to have a new look at several nuances of formulation of the 2D or 3D initial transport equations, as well. For example, a new unknown form of the radiative transport equation of the fourth-order is proposed for the case of the orthogonal scattering approach

Keywords

Scattering, absorption, light transport, radiative transport equations, two-dimensional problems, orthogonal scattering approach, pencil-like beam

Received 18.07.2017

Introduction. Much more real and close to realistic practical problems are spatial two-dimensional (2D) or three-dimensional (3D) scattering models. In the general case of the real 3D space, classic stationary Radiative Transport Equation (RTE) has a form with more numbers of independent spatial variables than it was used under 1D approximations. For the 3D space, besides *x, y, z* Cartesian coordinates, two spatial angles come into a play — polar angle Θ and azimuthal one ψ [1]:

$$\frac{dI(\tau,\mathbf{s})}{d\mathbf{s}} = -(\mu_a+\mu_s)I(\tau,\mathbf{s}) + \frac{\mu_s}{4\pi}\int_{4\pi} I(\tau,\ \mathbf{s}')p(\mathbf{s},\mathbf{s}')d\Omega. \tag{1}$$

As usual for RTE, here we denoted: $d\Omega = \sin(\theta) d\theta d\psi$ [sr] — the differential element of a solid angle, $I(\boldsymbol{\tau}, \mathbf{s})$ [W·sr^{-1}·m^{-2}] is the radiance depending in general both on the coordinate vector $\boldsymbol{\tau} = \{x, y, z\}$ and the directional unit vector $\mathbf{s} = \{s_x, s_y, s_z\}$ where $s_x^2 + s_y^2 + s_z^2 = 1$, $p(\mathbf{s}, \mathbf{s}')$ is the phase scattering function describing the scattering indicatrix from a direction $\mathbf{s}'$ to the direction $\mathbf{s}$. Phase scattering function obeys the normalization procedure:

$$\int\limits_{4\pi} p(\mathbf{s}, \mathbf{s}')\, d\Omega = 1. \tag{2}$$

The main difference of the RTE (1) from any ordinary differential equations before used consists in the necessity of integration of the scattered optical field over a spatial solid angle. This makes the problem much more complex. Another aspect is associated with the assessment of partial derivatives, because $\dfrac{dI(\boldsymbol{\tau}, \mathbf{s})}{d\mathbf{s}} = \mathbf{s}\,\mathrm{grad}[I(\boldsymbol{\tau}, \mathbf{s})]$ is the directional derivate. It is well-known that Eq. (1) is too common and too complex to be solved analytically in a closed form. Despite the long-term development of light transport and scattering theory (LT&ST), there are a few approximations only, for example, diffusion approximation [2–4], that allow the analytical solution in a number of practically important cases. In practice, a special requirement for such a solution is that it should describe at least backscattering radiance or a transmitted one on boundaries of the medium in the explicit closed analytical form, which allows an accurate and fast next solution of the inverse problem of the optics of turbid media, since the vast majority of diagnostic equipment, especially in biomedical optics [5], detect these radiation for the purpose to solve the inverse optical task, i. e., to obtain optical properties of the medium using the registered fluxes on the next step of data processing [6]. In the light of this statement, other approaches, apart from the exact analytical solutions, are not quite applicable. The Monte Carlo (MC) method of statistical simulations, for example, lacks clarity and consumes a lot of computing time that does not allow creating real-time diagnostic systems for medicine. The MC-based real time inverse optical task solution is impossible. In addition, it does not provide a solution in a form of a closed analytical expression, which could be easily analyzed for how one or another optical parameter of the medium of light propagation affects the final registered boundary radiation. The diffusion approximation, most popular today in biomedical optics, lacks the required precision near the frontal surface [1], i. e. it is not quite applicable for backscattering measurement techniques near the area of illumination. And so on. That is why the issue of finding new approaches to rigorous solutions of 3D problems is a subject of a study throughout all last decades [7–11 and others].

Reducing the RTE (1) to a 2D equivalent equation leaves not two, but three independent spatial variables, for example x, y, and θ. It simplifies the problem, but not so much. In this part of the article, due to the limited volume of it, we restrict ourselves to the problems of 2D cases only. Extension of the 2D ideas to a 3D problem consideration is not as difficult as the way from the 1D to a 2D level. Also, we will not bring the presentation to the level of a detailed description of full completed solutions of all possible 2D problems, as we did it for 1D problems, because our aim is to study the fundamentals of the theory, not to obtain final solutions for all applied tasks. We would rather like to understand better, how we could improve the theory, or how to find new approaches to solve the problem in the proper analytical way, than to solve any specific kind of problems in any way.

Any theoretical studies always are more effective and preferable when it starts from the simplest formulation of a problem and from the simplest approaches to find a solution. We have never came across previously in publications some detailed analysis of various simplest 2D models of scattering, like exist for 1D problems. Therefore, we decided to fill the gap by ourselves. For this part of the article, we have selected a number of the most interesting cases, which we considered as good examples to clarify several specialties existing in the multidimensional problem formulation. We hope they may encourage the emergence of new ideas and approaches in LT&ST. Also, we have to repeat that we are working in the field of biomedical optics [12], so examples and problems for turbid biological tissues are more habitual to us. We will use them in this part of the article, as previously. However, our results are also applicable to other light scattering media — clouds, milk glasses, turbid liquids, etc. They are not limited to biological tissues and media only.

Simplest 2D case without angular scattering. Bringing the gap between 1D and 2D problems, as the first step there is the reason to study the simplest extension of the 1D Kubelka — Munk (KM) model to a 2D level. Attempts of the extension of 1D KM approach to solve multidimensional problems have been made in a number of recent excellent studies [11]. Unlike the approaches considered, in this section to study fundamentals of the theory in detail we offer for consideration a particular case of the 2D turbid media with the absence of angular scattering. We assume the only presence of a backscattering process inside the 2D medium. This model is the most close to the initial KM approximation, and does not involve any integrals in the initial differential equations, that simplifies the solution very much. In the previous part of the article, we have shown the possibility of obtaining exact solutions based on the improved KM approach, so we can expect that in the 2D case it will be effective, as well.

Our schematic representation of the 2D turbid medium with scattering and absorption will still be as presented in Figures 1–2, part 2 [13], however the main differences now is that the medium has the second dimension "*Y*", as well as the incident radiation $f_0(y)$ is the 2D *flux density* with the dimension [W·m^{-1}], so the internal unknown functions are the flux densities $f_+(x, y)$ and $f_-(x, y)$ [W·m^{-1}], as well. Figure 1 explains the model.

Because of the absence of angular scattering, only a set of "straight light rays" ($f_+(x, y)$ and $f_-(x,y)$) will propagate in the medium. Therefore, the solution of the improved KM system (Eqs. (15) in Ref. [13]) is also valid in this model. However, the solution now is:

$$f_+(x, y) = C_1(y)e^{-\alpha x} + C_2(y)e^{\alpha x}; \quad f_-(x, y) = C_1(y)A_-e^{-\alpha x} + C_2(y)A_+e^{\alpha x}, \qquad (3)$$

where constants of integration C_i are now some functions of the second spatial coordinate y. They could be determined from the boundary conditions, for example:

$$f_+(0, y) = f_0(y) \text{ and } f_-(H_0, y) = 0.$$

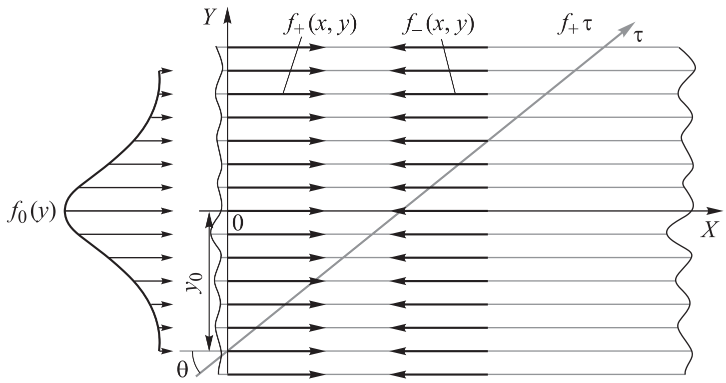

Fig. 1. Schematic representation of the 2D scattering problem without angular scattering

This simplest case without angular scattering is quite a good approach at single scattering approximation (SSA) inside strong absorbing media, where the long path length will lead to near full absorption of propagating radiation.

Remembering that second terms in the right-hand parts of the Eqs. (3) describe the reduction of the functions due to the limited medium length H_0, to simplify the solution we can consider a semi-infinite medium. It yields the solution in the simple form as follows:

$$f_+(x, y) = f_0(y)e^{-\alpha x}; \quad f_-(x, y) = f_0(y)A_-e^{-\alpha x}. \tag{4}$$

Why this result is interesting for us? It allows one to analytically derive any directional derivatives of $f_+(x, y)$ or $f_-(x, y)$ along any given vector $\boldsymbol{\tau}$ to test the general RTE (1). Is the general RTE valid for this 2D backscattering problem? If $|\boldsymbol{\tau}| = \tau$, then any spatial coordinates can be written as $x = \tau\cos\theta$, $y = \tau\sin\theta$, that yields, for example, for $f_+(\boldsymbol{\tau})$:

$$\frac{df_+(x, y)}{d\tau} = -\alpha f_0(y)\cos(\theta)e^{-\alpha x} + \frac{\partial f_0(y)}{\partial y}\sin(\theta)\, e^{-\alpha x}, \tag{5}$$

or, replacing the spatial variables (see Fig. 1):

$$\frac{df_+(\tau)}{d\tau} = -\alpha\cos(\theta) f_0(y_0 + \tau\sin\theta)e^{-\alpha\tau\cos\theta} + \frac{\partial f_0(y_0 + \tau\sin\theta)}{\partial(y_0 + \tau\sin\theta)}\sin(\theta)e^{-\alpha\tau\cos\theta}. \tag{6}$$

We see, that Eqs. (5), (6) quite differ from the general RTE (1). From the physical point of view, in this model, there is not any light power propagation in the direction $\boldsymbol{\tau}$, so we cannot formulate *a priory* a power loss along τ. However, as we have seen, the directional derivative Eq. (5) exists. Moreover, it is determined by the partial derivative of the incoming external flux density $\frac{\partial f_0(y)}{\partial y}$, that is not so evident from RTE (1). Since from the first equation of Eqs. (4) we have

$$f_+(\tau) = f_0(y_0 + \tau \sin\theta) e^{-\alpha\tau\cos\theta}, \tag{7}$$

then even in the case of the absence of scattering ($\mu_S = 0$) when $\alpha = \mu_a$, we cannot formulate Eq. (6) in the form

$$\frac{df_+(\tau)}{d\tau} = -\alpha f_+(\tau) = -\alpha f_0(y_0 + \tau\sin\theta)\, e^{-\alpha\tau\cos\theta}, \tag{8}$$

as it is required from RTE (1), because it is not correct in comparison with Eq. (6). Thus, the general RTE (1) is not valid for this simple scattering model, though the model is applicable for a description of real photometric measurement results. In our opinion, it proves that RTE is not applicable for all cases of scattering problems, as well as opens the way to seek new scattering models, which can have exact analytical solutions in the closed form of expressions like Eqs. (4), but which are not directly based on the classic RTE (1).

Pencil-like beam and a single scattering approximation. One more model, which has a quite simple representation and a closed-form solution, describes the single scattering approximation (SSA) for the pencil-like beam (δ-beam) which illuminates the 2D medium (Figure 2). In this model, the initial δ-beam F_0 [W] penetrates

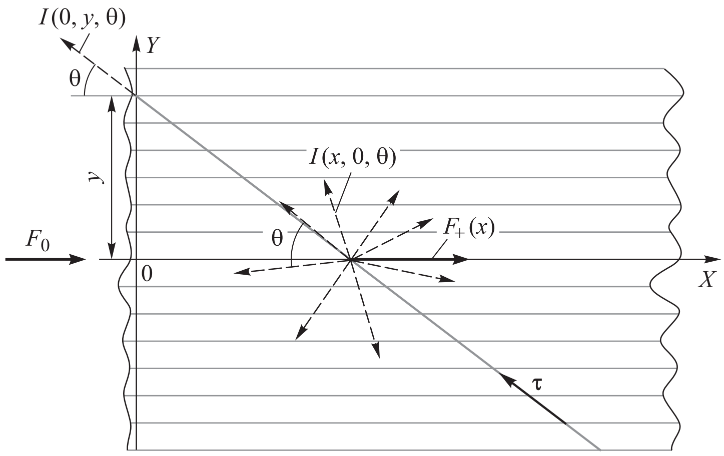

Fig. 2. Model of SSA for the pencil-like beam illuminating the 2D medium

the medium and propagates as a radiant flux $F_+(x)$ along the X-axis undergoing both scattering and absorption. At SSA, as we already know from 1D problems, a reduced radiant flux $F_+(x)$ has the form [14]:

$$F_+(x) = F_0 e^{-(\mu_a - \mu_\rho \ln(1-R))x}, \tag{9}$$

where R in this 2D model determines a total fraction of the radiant flux $F_+(x)$, which is scattered on each heterogeneity in all spatial directions along the flux's path inside the medium.

The scattered field is now the scalar field $I(x, y, \theta)$ of the radiant intensity or of the radiance with dimensions [W·sr^{-1}] or [W·sr^{-1}·m^{-2}] respectively, which for all 2D cases are reduced to [W·rad^{-1}] or [W·rad^{-1}·m^{-1}]. These dimensions differs from the dimension of the incident radiant flux F_0 or of the internal $F_+(x)$ [W], so for the transmitted radiation along the X-axis one need to consider *two components* of radiation: reduced radiant flux $F_+(x)$ and a series of appeared radiance or radiant intensities $I(x, 0, \pi)$ [1], which are scattered in the forward hemisphere. In general, this aspect is also missed in RTE (1).

The magnitude of $I(x, y, \theta)$ for each direction θ and for each single scatterer is determined by the 2D scattering phase function $p(\theta)$ [rad^{-1}], more exactly — by the product $Rp(\theta)$, if the phase function obeys the normalization procedure in the form of its integration in respect to the variable angle θ over the closed domain in the 2D space $[0, 2\pi]$:

$$\int_0^{2\pi} p(\theta)d\theta = 1. \tag{10}$$

It should be specially noted, that the magnitude of the radiant intensity $I(x,0,\theta) = Rp(\theta)F_+(x)$ along the X-axis reflects the scattering field for a point-like single source of radiation (a particle in the classic theory) lying on the X-axis. However, the total fraction of $F_+(x)$ scattered along Δx depends not only on parameters R and $p(\theta)$, but on the density of heterogeneities μ_ρ [m^{-1}], as well. There is a complex combined process, as we understand it now. Moreover, as it was shown in previous parts of the article [13, 15], the classic product $Rp(\theta)\mu_\rho$ [1] for description of this process is wrong. So to obtain $I(x, 0, \theta)$ for the elementary small dx containing a set of scatterers (particles), for example, for the backward hemisphere direction, one should first start the procedure for finding the limit:

$$\lim_{\Delta x \to 0} \frac{\Delta I(x,0,\theta)}{\Delta x} = \frac{\partial I(x,0,\theta)}{\partial x} = -\beta_2^*(\theta)F_+(x), \tag{11}$$

where $\beta_2^*(\theta)$ is the side-scattering coefficient [rad^{-1}·m^{-1}], which determines the angular scattering inside dx in the backward hemisphere. The right-hand part of the Eq. (11) is the 2D radiance [W·rad^{-1}·m^{-1}] of the element dx as of a secondary source of radiation. To avoid unnecessary detailing here, assume for this step $\beta_2^*(\theta) = \beta_2\rho(\theta)$, where β_2 is the backscattering coefficient at SSA in 1D problems [13]. After all, with the use of Eqs. (9) and (11), the solution for the radiant intensity $I(x, y, \theta)$ at any point of this 2D turbid media should not pose any serious difficulties. It is necessary only to take into consideration the exponential attenuation of $I(x, 0, \theta)$ along its next pathway after a formation on the X-axis.

For example, the scattered radiant intensity $I(0, y, \theta)$ escaping the frontal surface of the medium, excepting the backscattered one $I(0, 0, 0)$, in this case has the closed analytical form, comparable to the Ref. [13], Eq. (8):

$$I(0,y,\theta)=F_0\frac{\beta_2^*(\theta)}{2\mu_a+\mu_\rho\ln(1-R)}e^{-(\mu_a-\mu_\rho\ln(1-R))y\frac{\cos\theta}{\sin\theta}}e^{-\mu_a\frac{y}{\sin(\theta)}}=$$

$$=F_0\frac{\beta_2^*(\theta)}{2\mu_a+\mu_\rho\ln(1-R)}e^{-\left(\mu_a(\cos\theta+1)-\mu_\rho\cos\theta\,\ln(1-R)\right)\frac{y}{\sin\theta}}. \qquad (12)$$

For any extreme cases of $\theta\to 0$ at $y\neq 0$ one can obtain $I(0, y, 0) = 0$. However, for the extreme case of $\theta = 0$ at $y = 0$, the $I(x, 0, 0)$ exists. There are a variety of heterogeneities along X-axis, which scatter radiation in the back direction ($\theta = 0$). Using Eq. (8) and Eq. (11) of the part 2 [13], for semi-infinite medium one can find for $I(0, 0, 0)$ the similar equation, as it was obtained for $F_-(0)$ in the 1D case, but accurate to the accepted value of $\beta_2^*(\theta)$. The main difference is only appeared in dimensions of quantities ([W] and [W·rad^{-1}]) due to the angle scattering in the 2D case. The similar approach should be applied to calculate the forward-scattered radiant intensity $I(x, 0, \pi)$.

In our opinion, the result Eq. (12) is remarkable in two ways. First, since it has a closed-form analytical expression, it can be theoretically analyzed as it is shown, for example, in Fig. 3.

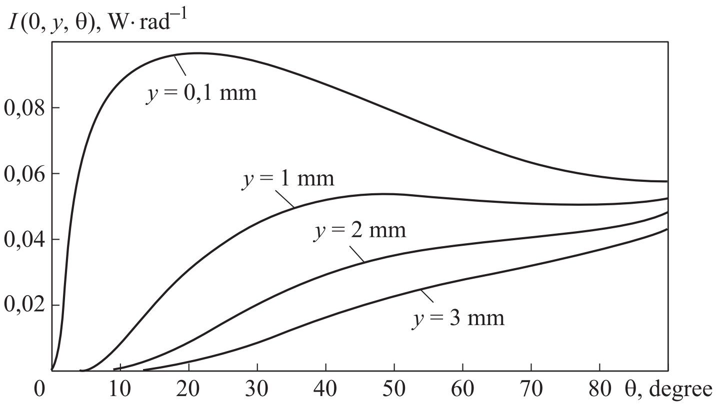

Fig. 3. 2D Radiant intensity $I(0, y, \theta)$ for semi-infinite 2D medium at F_0 = 1 W, R = 0.4,
$\mu_a = 0.1$ mm^{-1}, $\mu_\rho = 0.5$ mm^{-1}, $p(\theta)=\frac{1}{3\pi}(1+\cos^2\theta)$

One can see, that for comparable short distance "y" from the point of illumination ($y < 1$ mm), the radiant intensity $I(0, y, \theta)$, if $y \neq 0$, has quite uniform distribution per angle θ for the given Rayleigh phase function with a local maximum at small angles $\theta < 30°$. The more distance we take into consideration, the more magnitude of the intensity is shifted to larger angles. This dependence is a function of R, μ_a, μ_ρ, and $p(\theta)$, so these optical properties can be studied by means of angular measurements of the radiant intensities $I(0, y, \theta)$ escaping the medium at large distances from the point of illumination. It should be remembered only, that any measurements are carried out in the receive aperture $\Delta\theta$ (practically, it is impossible to measure pure $I(0, y, \theta)$), so the integration of $I(0, y, \theta)$ over the aperture angle θ is required.

Then, since Eq. (12) is a closed-form analytical expression, like it was done in the previous section, we can try to derive a directional derivate for each $I(x, y, \theta)$ for the purpose of comparison of the equation with the general RTE (1). If, for instance, we select a direction $\boldsymbol{\tau}$ in Fig. 2 from the right lower corner up to the left top corner, and schematically depict the intensity $I(\boldsymbol{\tau}) = I(x, y, \theta)$ as a function of $\boldsymbol{\tau}\sin\theta$, we get the following approximate picture (Figure 4).

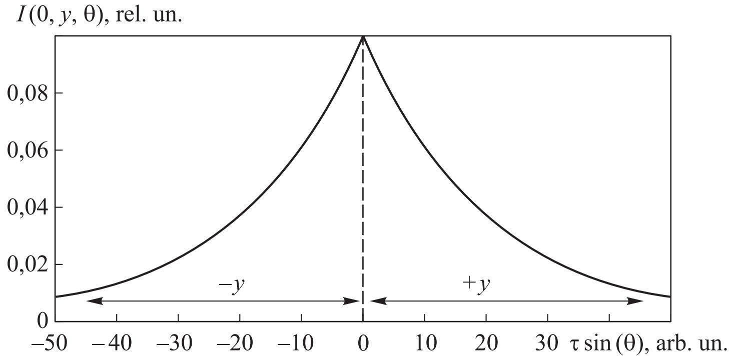

Fig. 4. Formation of a fracture point for a curve $I(\boldsymbol{\tau})$ at crossing the line $y = 0$ due to the pencil-like beam illumination and SSA

Once again we see the non-differentiable function at the point $\boldsymbol{\tau}\sin\theta = 0$, so the general RTE for $I(\boldsymbol{\tau})$ is not valid in this point. To avoid the line $y = 0$, i. e., the X-axis, we need to formulate two similar differential equations separately for both a left-hand side and a right-hand side $I(\boldsymbol{\tau})$ together with corresponding conjugation conditions along the X-axis where $y = 0$. Thus, we have found out, that for such a simple formulation of the 2D problem, we should define a number of differential equations, at least four if to take into consideration the specialties of formation of $I(0, 0, 0)$, $I(0, 0, \pi)$, and $F_+(x)$. There is not a single equation to solve the problem properly for all quantities, which are interesting to us.

Approximation of the orthogonal scattering. Now we are close to the question, how other forms of RTE, more precisely describing the problems, can be formed in a number of practically important cases. Previously it was shown, that another form of RTE could exist for a slab geometry [8]. This section proposes a new approach for the 2D orthogonal-scattering model (OSM) [14]. We will use the multi-fluxes KM approach, similar to [7], with optical properties of the medium in the definition of Eqs. (17a), (17b), part 2 [13]. It is assumed a 2D OSM, i. e., in both "x" and "y" direction two *flux densities* [W·m^{-1}] propagate only, forward $f_{i+}(x, y)$ and backward $f_{i-}(x, y)$ ones (Figure 5), where $i = x$ or $i = y$. In the medium, the light scattering takes place as the backscattering or orthogonal-scattering ($f_x(x, y) \leftrightarrow f_y(x, y)$) events only.

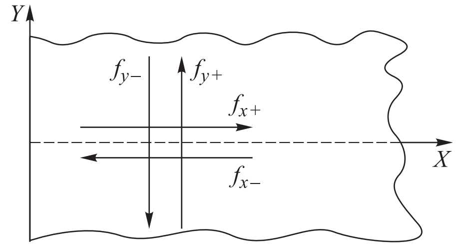

Fig. 5. Fluxes inside the turbid tissue for OSM

It gives the following system of four partial differential equations:

$$\begin{aligned}
\frac{\partial f_{x+}(x,y)}{\partial x} &= -\beta_1 f_{x+}(x,y) + \beta_2 f_{x-}(x,y) + \beta_3 (f_{y+}(x,y) + f_{y-}(x,y));\\
\frac{\partial f_{x-}(x,y)}{\partial x} &= \beta_1 f_{x-}(x,y) - \beta_2 f_{x+}(x,y) - \beta_3 (f_{y+}(x,y) + f_{y-}(x,y));\\
\frac{\partial f_{y+}(x,y)}{\partial y} &= -\beta_1 f_{y+}(x,y) + \beta_2 f_{y-}(x,y) + \beta_3 (f_{x+}(x,y) + f_{x-}(x,y));\\
\frac{\partial f_{y-}(x,y)}{\partial y} &= \beta_1 f_{y-}(x,y) - \beta_2 f_{y+}(x,y) - \beta_3 (f_{x+}(x,y) + f_{x-}(x,y)),
\end{aligned} \tag{13}$$

where β_1, β_2, β_3 are attenuation, backscattering, and lateral-scattering coefficients respectively. This system can be resolved analytically with the use of the standard technique of multiples derivatives and of reduction of the system to one differential equation of the fourth-order. The first step, for example, for the system (13) in this way is to differentiate the first and the second equations for y, and the third and the fourth equations for x. Then, by a substitution of the initial equations for the corresponding derivatives of the first-order into the new system of the second-order equations, one can reduce the number of unknown functions. Repeating the process three times, leaving only, for example, $f_{x+}(x, y)$, it is not so difficult to obtain the final RTE for OSM in the form:

$$\frac{\partial^4 f_{x+}(x,y)}{\partial x^2 \partial y^2} - \alpha^2 \left[\frac{\partial^2 f_{x+}(x,y)}{\partial x^2} + \frac{\partial^2 f_{x+}(x,y)}{\partial y^2} \right] + \left[\alpha^4 - 4\beta_3^2 (\beta_1 + \beta_2)^2 \right] f_{x+}(x,y) = 0, \tag{14}$$

where, as previously, $\alpha = \sqrt{\beta_1^2 - \beta_2^2}$. As one can see, this form is much differ from the classic RTE, but is more reasonable. Comparing to the classic KM equations, we have to note, that the more directions of light propagation we consider, the more order of the final differential equation we have. Why the standard RTE is the single first-order differential equation, if even in the case of 1D approach, we have the system of two differential equations, so that the final equation is of the second order for each flux [13]? In our opinion, the standard RTE (1) contains the main draft heuristic idea about the ray changing along the path of its propagation, but it is not enough to solve the problem. To solve the problem, it is necessary to formulate the problem in the correct completed form.

The analytical partial solution of the Eq. (14) for each fluxes is [15]:

$$f_i(x, y) = (C_{i1}e^{tx} + C_{i2}e^{-tx})(C_{i3}e^{ty} + C_{i4}e^{-ty}), \qquad (15)$$

where C_{ik} — constants, which depend on boundary conditions, and $t = \sqrt{\alpha^2 + 2\beta_3(\beta_1 + \beta_2)}$. In this solution, the main problem is still the same — non-differentiable equations in some points of the medium. For example, for the pencil-like illuminating beam the radiative flux density distribution in this model is still piecewise (stepwise) with indefinite derivatives in breaking points of the first order along the line $y = 0$ (Figure 6).

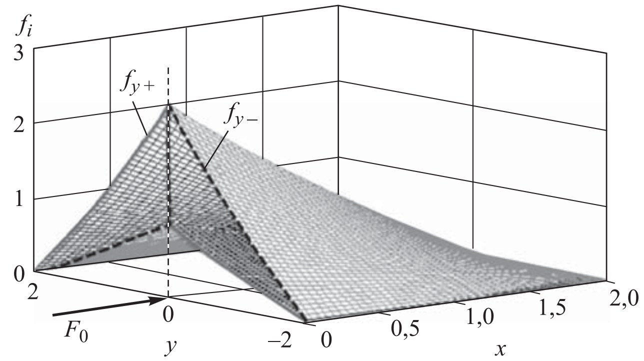

Fig. 6. Radiative flux density distribution in the 2D model of orthogonal scattering (OSM)

What is also important in the 2D OSM problem — it is the stepwise character of the breaking points in the macroscopic scale of the medium. If in 1D formulation we have seen the piecewise character in the microscopic scale of the turbid medium due to the existence of heterogeneities inside the medium [13, 15], then here we saw the same problem on the macro-level, but due to the nuances of external illumination. Therefore, the problem of the proper boundary condition formulation is also very important. It can be resolved, for example, with the use of the rough surface diffraction technique [16, 17], but it is a completely other problem.

General conclusion. The light transport and scattering theory (LT&ST) is widely used today in biomedical optics, ocean optics, optics of atmosphere, etc. However, it is well-known that there are difficulties with the analytical and closed-form solution of the classic radiative transport equation (RTE). In general, RTE does not have the analytical solution. As we have reported in our previous publications, the problem particular follows from the not quite correct formulation of main equations in LT&ST for a number of applications. In this article, we tried to prove some of these assumptions from "first principles", describing, first, step by step, a number of different models of 1D problems. In this way, it was shown that the scattering coefficient is rather a parameter of the approach used, of the approximation, than of the optical properties of the medium. It depends not only on real optical properties of a turbid medium, but also on the mathematical formulation of the problem. The scattering coefficient is the parameter, which was introduced by researchers to describe the scattering process as a continuous photometric process using appropriate mathematical tools. Therefore, in different tasks it can vary and needs an accurate definition. Based on this result, the 1D Kubelka — Munk (KM) approach was cardinally improved. It was shown, that there are a number of tasks where we cannot separate absorption and scattering coefficients (K and S in the KM notations and μ_a and μ_s in the RTE ones) in the extinction coefficient in RTE. Such a superposition of scattering and absorption coefficients was a direct consequence of the accepted phenomenological formalism about independence of absorption and scattering processes. It was not proved previously, just theoretically assumed and accepted. However, now we have rigorously proved that absorption and scattering processes are not independent. It allowed us to improve the theory. More correct formulation of the extinction coefficient, for example, in application to the classical two-flux Kubelka — Munk (KM) approach, which is a good 1D limit for the general RTE, allowed us to obtain the exact analytical solution for boundary radiant fluxes (backscattered and transmitted ones), contrary to the classic KM approximation. These fluxes are registered by diagnostic equipment in experiments, in that number in biomedical applications, so this result is very important for the practical usage. In addition, such a result leads to a necessity to revise a number of basic definitions of optical properties of turbid media. Scattering coefficient and albedo are the most important in this sense. It was visible shown in the paper that the magnitude of albedo can differ for the classical and for the offered new definitions.

Development of all these ideas and approaches for two (2D) or more dimensional problems opened the way to have a new look at several nuances of formulation of the 2D or 3D initial transport equations. For example, new form of RTE (14) was derived in the study for the case of orthogonal scattering approximation. It is the fourth-order differential equation contrary to the classic RTE, and it has a closed form exact analytical solution. The classic RTE is shown in the study to be not applicable for all cases of 2D scattering problems. It proves the statement that the standard RTE (1) contains the main draft heuristic idea about the ray changing along the path of its propagation only, but it is not enough to solve the problem. To solve the problem, it is necessary to formulate the problem in the correct completed form. In a total, in our opinion, such a look offered allows one to understand better some fundamentals of LT&ST, for example, a mechanism of formulation of accurate initial equations for different problems, finding out several new ways to improve and to develop the theory. Mainly, it highlights the way for the paradigm switching in LT&ST, the way to jump from the search for new solutions and approximations for original RTE to the search for and to the formulation of new closed systems of equations, which accurate describe a problem in the completed form, and which have an exact analytical solution.

REFERENCES

[1] Ishimaru A. Wave propagation and scattering in random media. New York, London, Academic Press, 1978. 572 p.

[2] Davison B. Neutron transport theory. Oxford University Press, 1957. 450 p.

[3] Case K.M., Zweifel P.F. Linear transport theory. Addison-Wesley Publ. Co., 1967. 342 p.

[4] Farrell T.J., Patterson M.S., Wilson B. A diffusion theory model of spatially resolved, steady-state diffuse reflectance for the noninvasive determination of tissue optical properties *in vivo*. *Med. Phys*., 1992, vol. 19, no. 4, pp. 879–888. DOI: 10.1118/1.596777

[5] Tuchin V.V. Handbook of optical biomedical diagnostics PM107. SPIE Press, 2002. 1110 p.

[6] Rogatkin D.A. Basic principles of organization of system software for multifunctional noninvasive spectrophotometric diagnostic devices and systems. *Biomedical Engineering*, 2004, vol. 38, iss. 2, pp. 61–65. DOI: 10.1023/B:BIEN.0000035722.72246.bf

[7] Yoon G., Welch A.J., Motamedi M., Van Gemert M.C.J. Development and application of three-dimensional light distribution model for laser irradiated tissue. *IEEE J. Quant. Electr*., 1987, vol. 23, no. 10, pp. 1721–1733.

[8] Rogatkin D.A. An approach to the solution of multidimensional problems of the theory of light scattering in turbid media. *Quantum Electronics*, 2001, vol. 31, no. 3, pp. 279–281. DOI: 10.1070/QE2001v031n03ABEH001932

[9] Liemert A., Kienle A. Light transport in three-dimensional semi-infinite scattering media. *J. Opt. Soc. Am. A*, 2012, vol. 29, iss. 7, pp. 1475–1481. DOI: 10.1364/JOSAA.29.001475

[10] Machida M. Singular eigenfunctions for three-dimensional radiative transport equation. *J. Opt. Soc. Am. A*, 2014, vol. 31, iss. 1, pp. 67–74. DOI: 10.1364/JOSAA.31.000067

[11] Sandoval C., Rim A.D. Extending generalized Kubelka — Munk to three-dimensional radiative transfer. *Applied Optics*, 2015, vol. 54, iss. 23, pp. 7045–7053. DOI: 10.1364/AO.54.007045

[12] Rogatkin D., Shumskiy V., Tereshenko S., Polyakov P. Laser-based non-invasive spectrophotometry — an overview of possible medical application. *Photonics and Lasers in Medicine*, 2013, vol. 2, no. 3, pp. 225–240.

[13] Persheev S., Rogatkin D.A. A new look at fundamentals of the photometric light transport and scattering theory. Part 2: One-dimensional scattering with absorption. *Vestn. Mosk. Gos. Tekh. Univ. im. N.E. Baumana, Estestv. Nauki* [Herald of the Bauman Moscow State Tech. Univ., Nat. Sci.], 2017, no. 6, pp. 65–78. DOI: 10.18698/1812-3368-2017-6-65-78

[14] Guseva I.A., Tarasov A.P., Rogatkin D.A. New form of the transport equation for the case of 2D orthogonal scattering approximation in biooptics. *Proc. 2016 Int. Conf. Laser Optics*, 2016, pp. S2–23.

[15] Persheev S., Rogatkin D.A. A new look at fundamentals of the photometric light transport and scattering theory. Part 1: One-dimensional pure scattering problems. *Vestn. Mosk. Gos. Tekh. Univ. im. N.E. Baumana, Estestv. Nauki* [Herald of the Bauman Moscow State Tech. Univ., Nat. Sci.], 2017, no. 5, pp. 78–94. DOI: 10.18698/1812-3368-2017-5-78-94

[16] Rogatkin D.A. Scattering of electromagnetic waves by a randomly rough surface as a boundary problem of laser radiation interaction with light-scattering materials and media. *Optics and Spectroscopy*, 2004, vol. 97, iss. 3, pp. 455–463. DOI: 10.1134/1.1803651

[17] Giddings T.E., Kellems A.R. Analytical model for radiative transfer including the effects of a rough material interface. *Applied Optics*, 2016, vol. 55, iss. 24, pp. 6606–6616. DOI: 10.1364/AO.55.006606

Persheyev S. — Dr. Sc. (Eng.), Research Fellow, School Physics and Astronomy, St Andrews University (College Gate, St Andrews KY16 9AJ, Fife, Scotland, UK).

Rogatkin D.A. — Dr. Sc. (Eng.), Assoc. Professor, Head of Medical and Physical Research Laboratory, Moscow Regional Research and Clinical Institute named after M.F. Vladimirsky (Shepkina ul. 61/2, Moscow, 129110 Russian Federation).

Please cite this article as:

Persheyev S., Rogatkin D.A. A New Look at Fundamentals of the Photometric Light Transport and Scattering Theory. Part 3: Bridges to Multi-Dimensional Problems. *Vestn. Mosk. Gos. Tekh. Univ. im. N.E. Baumana, Estestv. Nauki* [Herald of the Bauman Moscow State Tech. Univ., Nat. Sci.], 2018, no. 2, pp. 60–72. DOI: 10.18698/1812-3368-2018-2-60-72